THE INSTITUTE OF PAPER CHEMISTRY

Appleton, Wisconsin

END-LOAD BOX COMPRESSION

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A Preliminary Report

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SUMMARY

An analysis of the end-to-end compression strength of RSC boxes was carried out, utilizing the theory of the ultimate compressive strength of plates. An equation was derived which relates maximum box load to (a) the edgewise compression strength of the combined board in the machine direction, (b) the flexural stiffnesses of the combined board in its principal directions, and (c) the length, width, and depth dimensions of the box. The empirical constants of the equation were evaluated from data on 57 samples of commercially-produced boxes fabricated with kraft liners, A, B, and C-flute construction, various series, and a range of dimensions.

On the average, the estimated box loads differed from the observed loads of the 57 samples by 8-1/4%. With one exception, none of the differences exceeded 18%.

The end-load formula is in qualitative agreement with the following major characteristics of end-load compression:

a. With given components, B-flute boxes of a given size test higher than C-flute; and, in turn, C-flute boxes test higher than A-flute.

b. End-load compression strength decreases as the gap between inner flaps increases, that is, as the length dimension of the box increases relative to the width dimension.

c. The flap panels are the primary load-carrying panels in end-to-end compression.

For the purpose of refinements in the analysis, and its application, work is underway with regard to methods of evaluating the compression strength properties of combined board and components which govern end-load box compression. It is recommended that further investigation be undertaken with respect to analysis of (a) an all-flaps-meet style of box, (b) the effective flexural stiffness of the flap panels, and (c) the differing behavior of flap panels and side panels.

INTRODUCTION

Although top-to-bottom compression has probably received greater attention in the evaluation of the quality of corrugated boxes, in many instances the end-to-end compression strength is fully as important. For example, end-load strength may give useful information with regard to the ability of the box to withstand forces incurred during accelerations and decelerations of a moving vehicle or on a conveyor. Although the dynamics of this type of service hazard are not simulated in the laboratory end-load compression test, this type of test may be expected to give some indication of the ability of the container to withstand this hazard. From an analytical standpoint, the ability to predict the static end-to-end compression strength of a box from consideration of component quality and box design would seem to be a prerequisite to adequately predict dynamic effects.

The object of the present study is to develop an equation relating end-to-end compression strength of RSC boxes to combined board (or component) properties and box dimensions. It is hoped that the equation will have simplicity and application comparable to the formula which has been evolved for top-load box compression (1). The results of this work may be expected to find application in the definition and required magnitudes of those properties of linerboard and medium which are important to good box performance.

A brief summary of recent work in the analysis of end-load compression was presented in a preliminary report to the Technical Committee, dated June 18, 1963 (2). The present report contains a more complete description of the work.

ANALYSIS OF BOX BEHAVIOR IN END-LOAD COMPRESSION

Unlike top-load compression, the load-bearing panels in end-to-end compression are of two distinctly differing types. One pair, such as ABCD in Fig. 1, is composed of flaps and may be termed flap panels. The other pair (BCEF) is composed of single-wall board with flutes "horizontal" and may be termed side panels. It may be anticipated that an end-load compression formula will involve the sum of two terms—one term pertaining to each of the types of panels.

With regard to the flap panels, it has been observed that failure almost always originates in the combined board of the outer flaps in the area between the inner flaps. Failure starts at the edge, G, of the outer flaps, and rapidly progresses to the edges (AD and BC) of the flap panel and then into the adjacent side panels. The failure apparently occurs initially in the single-face liner, because this liner is on the concave side as the panel bows outward, and, hence, is the more highly stressed liner. Failure of the single-face liner is a compressive buckling of the liner between flute tips.

Immediately after the single-face liner fails, the flap panel may be observed to snap back from its bowed shape to a nearly plane configuration. Coincident with this snap-back is compressive failure of the double-face liner. The mechanism of the snap-back is believed to be as follows. Prior to failure of the single-face liner, elastic energy due to flexure is stored in the inside flaps while the panel bows outward. When the single-face liner fails and thereby loses virtually all of its compressive resistance, the stored energy in the inner flaps is recovered and is sufficient to force the panel back to its plane configuration. In so doing, additional compressive stress is brought onto the double-face liner of the outer flap, causing it to fail by compressive buckling between the flute tips.

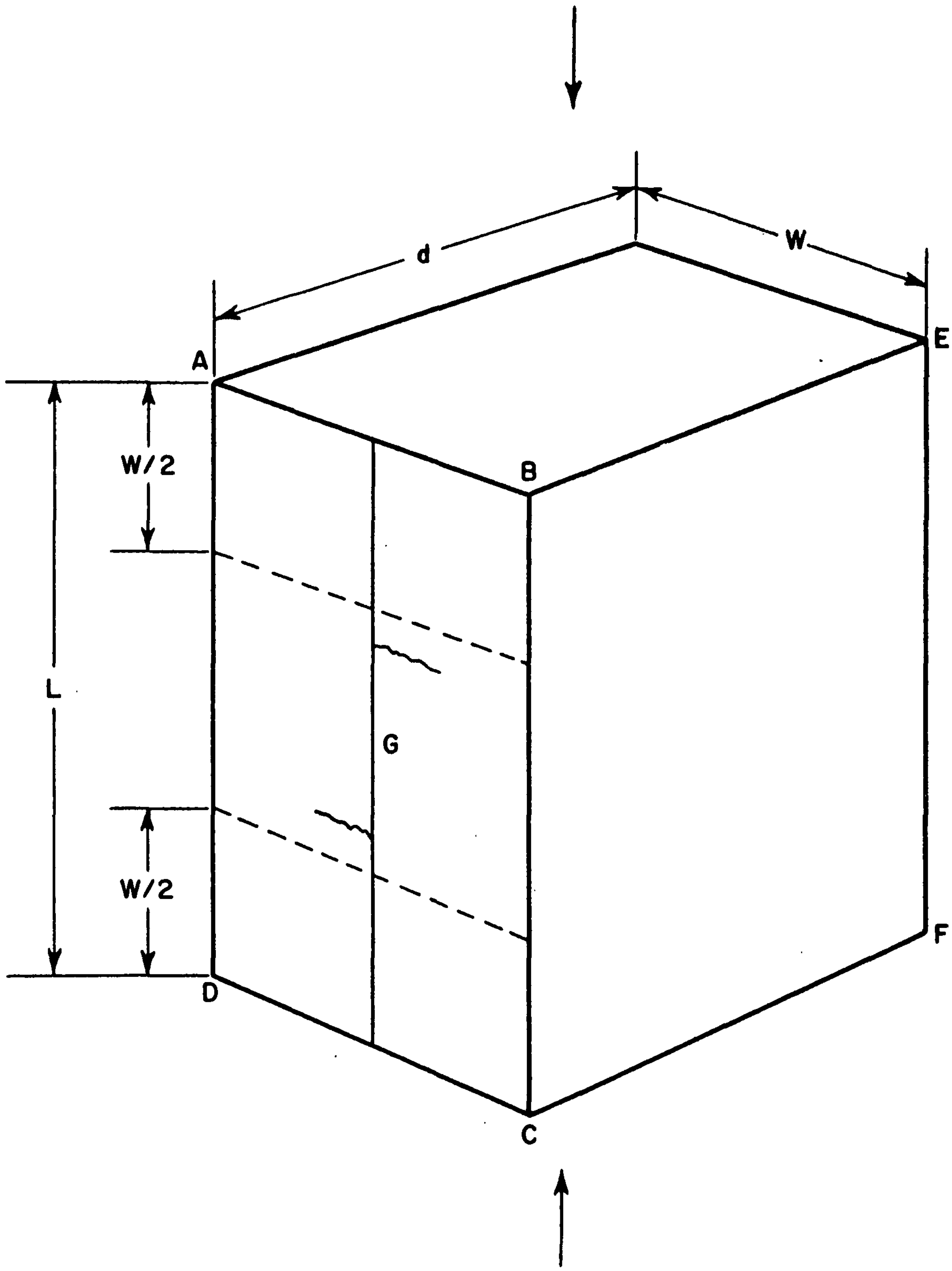

Figure 1. RSC Box in End-to-End Load Orientation

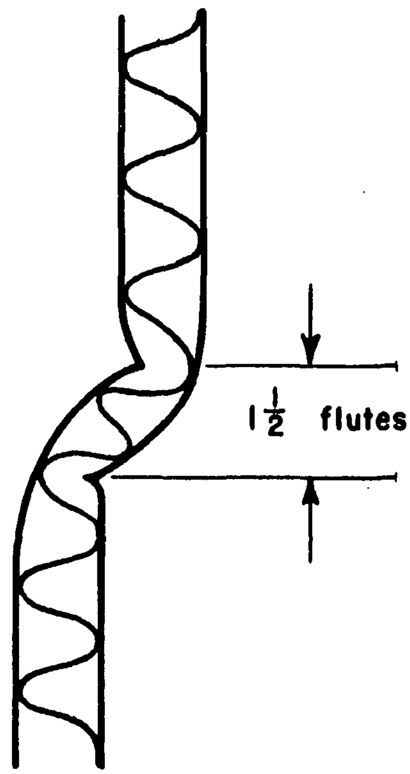

Figure 2. Typical Failure of Combined Board in Outer Flap in End-Load Compression

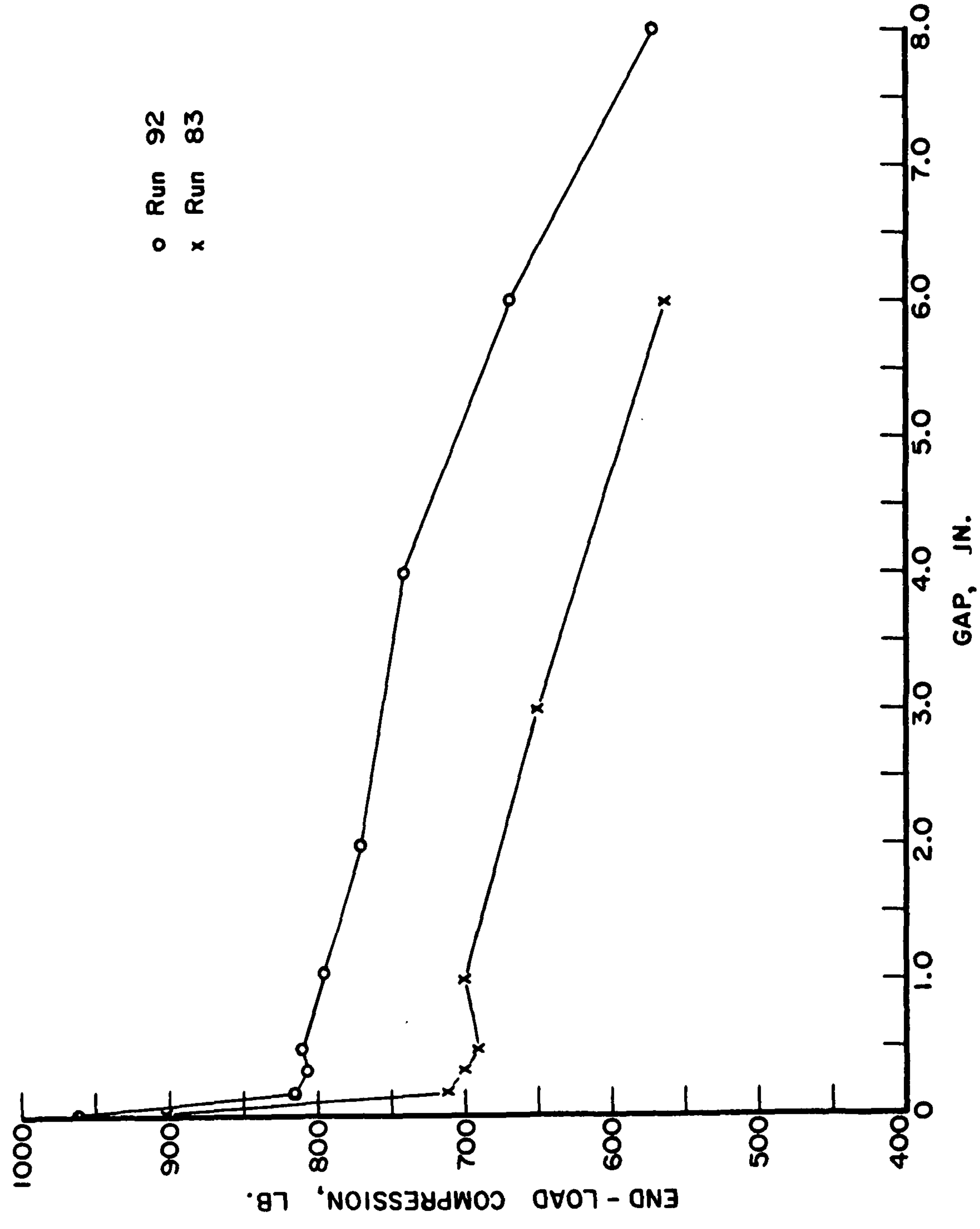

Figure 3. Relationship Between End-Load Compression Strength and Gap Between Inner Flaps in 12-Inch Cubical Boxes Having Inner Flaps Cut Away by Varying Amounts (Run 83, 42-Lb. Liners; Run 92, 52-Lb. Liners; 33-Lb. Semichemical Medium in Both)

TABLE I

THE EFFECT OF FLAP-GAP ON THE END-LOAD STRENGTH OF CUBICAL BOXES

Spec. No.	Box Load, lb. Gap, in. 0	1/6	1/3	1/2	1	2	3	4	6	8
	Run 83 (42-lb. liners)									
1	930	710	608	670	630		659		565	
2	901	739	745	700	685		612		517	
3	1046	581	714	650	726		650		595	
4	816	751	720	714	699		648		579	
5	726	735	710	684	672		674		598	
6	937	716	647	674	735		703		582	
7	891	749	606	664	619		626		457	
8	910	738	769	560	659		679		566	
9	934	710	742	781	676		672		600	
10	947	692	744	719	736		642		571	
11	892	--	703	700	744		562		452	
12	840	--	697	693	705		520		514	
13	958	--	--	782	743		732		622	
14	--	--	--	--	747		717		672	
15	--	--	--	--	734		--		563	
Av.	902	712	700	692	701		650		564	
	Run 92 (52-lb. liners)									
1	1059	750	734	887	904	882		888	636	604
2	922	842	850	664	774	704		682	581	503
3	1008	924	824	804	868	809		885	785	594
4	1100	821	764	897	801	805		674	560	582
5	807	740	864	798	865	697		697	660	510
6	813	--	--	--	836	696		674	598	579
7	819	--	--	--	816	791		743	712	571
8	967	--	--	--	729	724		797	768	606
9	922	--	--	--	736	816		616	630	556
10	1050	--	--	--	766	773		638	688	528
11	988	--	--	--	588	865		753	598	587
12	1008	--	--	--	801	630		806	657	616
13	1013	--	--	--	797	794		752	690	569
14	--	--	--	--	854	792		--	770	--
Av.	960	815	807	810	795	770		739	667	570

For example, with a small gap of 1/6 inch, which might be incurred in improperly sealing an all-flaps-meet box, the end-load strength dropped from 902 to 712 lb. in the case of the boxes with 42-lb. liners and from 960 to 815 lb. for the boxes with 52-lb. liners. Thus, the butting of the inner flaps apparently makes a major contribution to end-load strength. In effect, the butting causes the flap panel to approach more nearly double-wall combined board, whereby some load is supported by the butted inside flaps. Even a very small gap, on the other hand, necessitates passing all the compression load on the panel through a zone of single-wall board, thereby limiting the load-carrying capacity of the flap panel to the strength of the single-wall board.

There is some question as to the most effective method of evaluating the strength of the combined board or components with respect to their resistance to the type of failure that triggers end-load box failure. Strength in the machine direction of the board is involved, of course, but there are at least three test methods which may be considered. They are as follows:

1. Short column of combined board. This type of test appears to have the attribute that it introduces the flute width effect which is intrinsically involved in the buckling of the liners between flute tips in the gap of the box. Also, this method can be expected to account for the effect of "washboarding" of the single-face liner which probably increases its susceptibility to buckling. On the debit side, the short column test subjects both the single-face and double-face liners to essentially the same compressive stress (inasmuch as the specimen does not bow), while in the general case of end-load box compression, the flap panel bows outward and the single-face liner is more highly stressed. The result should be that the short column test overestimates the load which the combined board in the gap is capable of supporting in the end-load orientation of the box.

2. Long column of combined board. A long column test may be conducted in a manner so that the column is forced to bow with the single-face liner on the concave side. An attribute of this type of test is that the single-face liner is caused to buckle between flute tips before the double-face liner fails, in similitude of the behavior of the outer flap in end-load box compression. A disadvantage of this type of test is that a long column load is not a basic material property, but rather depends upon the height of the column specimen. It would be necessary, therefore, to determine the relationship between the column length of the test specimen and the effective length of the board in the flap of the box (the latter is probably a length intermediate between the gap length and the length of the outer flap). Even if this were accomplished, however, the long column strength could be expected to underestimate the load-carrying ability of the board in the box because the long column test does not introduce the resistance to bowing from transverse stresses owing to the support from the lateral edges of the flap panel (AD and BC of Fig. 1).

3. Edgewise compression strength of the single-face liner. Inasmuch as end-load box failure is triggered by failure of the single-face liner between flute tips, it may be suggested that an appropriate evaluation of the board could be achieved by an edgewise compression test on the single-face liner alone (say, by the modified ring compression test). This type of evaluation, however, would not account for the effect of flute size, namely, that with given components, B-flute boxes have higher end-load strength than C-flute boxes; and C-flute, in turn, higher than A-flute. [It may be noted that another combined board property, presumably entering into end-load behavior, namely, flexural stiffness, exhibits the reverse trend for A, C, and B-flute (1), and thus cannot account for differences in end-load strength between flute sizes.] It would appear, therefore, that if end-load strength is to be related to a property of the single-face liner by

itself, then this property should be the buckling strength of the liner rather than its ultimate edgewise compression strength. The liner buckling strength can be expected to depend upon the column height (that is, flute width), degree of fixity at the flute tips, flexural stiffness of the liner in the machine direction, and the initial flatness of the liner (that is, degree of "washboarding").

Considering the pros and cons of the three types of test properties discussed above, it is believed that none, by itself, is ideally suited for describing the strength of the material in end-load compression. In the preliminary studies concerned with constructing an end-load formula, the short column test, coupled with the flexural stiffnesses of the combined board, was used. Exploratory studies are now being conducted with the long column test method. The buckling strength of the single-face liner (<u>not</u> its ultimate edgewise compression strength) is perhaps the most attractive property from a theoretical standpoint, but will require development of a special test or method of estimation.

Turning to consideration of the side panels, it has been observed from load distribution measurements that the perimeter scorelines, BE and CF of Fig. 1, roll and crush in advance of maximum box load and support reduced loads at the time the box reaches its maximum load (3). The average load supported by a side panel is only about 1/4 to 1/3 of the average load supported by a flap panel. These observations indicate that the side panels make only a modest contribution to the total box load when the width and depth dimensions of the box are of comparable magnitude. Furthermore, the load supported by the side panel is largely associated with the resistance to rolling and crushing of the board near the scoreline at the loading perimeter (panel scores of the box).

By way of summary, the following observations on the behavior of corrugated boxes in end-load compression should be accounted for in the development of a formula relating compression strength to combined board or component properties and box dimensions:

a. With given components, B-flute boxes of a given size test higher than C-flute and, in turn, C-flute tests higher than A-flute.

b. End-load compression strength decreases as the gap between inner flaps increases. A major loss in strength occurs between the case of zero gap (inside flaps butt) and a small gap of 1/6 inch.

c. The flap panels are the primary load-carrying panels in end-to-end compression.

PRELIMINARY DEVELOPMENT OF AN END-LOAD COMPRESSION FORMULA

As a first approach in developing a formula for end-load box compression, it was decided to apply the methods which led to the simplified formula for top-load compression (4). The latter development, it may be recalled, started from a semi-empirical equation for a plate loaded by edgewise compression forces and then utilized a number of approximations to obtain a simplified equation relating box load to edgewise crushing strength and flexural stiffnesses of the combined board and box perimeter.

If this same general approach is applied to the flap panels (actually to each outside flap individually), there results the following expression for the maximum load, P_f, supported by the two flap panels:

$$P_f = c\, P_{mx}^{\ b} \left(g\sqrt{D_x D_y}\right)^{1-b} W^{2b-1} \quad (1)$$

where P_{mx} = compression strength of the combined board in the machine direction (evaluated by a short column with wax-reinforced loading edges lb./in.)

$\sqrt{D_x D_y}$ = composite flexural stiffness of combined board, lb. - in.

g = a function of box dimensions, as discussed below

W = box width, in.

a,b,c = empirical constants

The major innovation needed in applying the ultimate plate strength theory to end-load compression is with regard to the effective flexural stiffness of the flap panels. This stiffness is no longer simply $\sqrt{D_x D_y}$, as in the case of top-load compression, but is a more complex parameter to account for the fact that a portion of the flap panel, in the general case, is of double thickness of combined board and the remainder (in the gap) is of single thickness.

An attempt to overcome this difficulty was by multiplying $\sqrt{D_x D_y}$ by a factor, g, which is a function of the relative amount of double and single thicknesses of combined board in the flap panel. A plausible form for g is

$$g = (1 + \frac{W}{L})^e, \tag{2}$$

where, L, is the length of the box, W, is the width, and e is a numerical constant. Reference to Fig. 1 will show that W/L is the ratio of the height of the double-thickness board to the total height of the panel. It may be reasoned that as the width of the box approaches the length dimension, the flap gap decreases and the effective flexural stiffness of the panel increases. The effective stiffness should not exceed about 4.0 times the single-wall stiffness, however, as this value would correspond approximately to the stiffness of two single-wall boards laminated together. At the other extreme, as the box width becomes small relative to the height of the flap panel, the flap gap increases and the effective flexural stiffness approaches that of a single thickness of combined board. Both of these extremes can be approximately satisfied by the function of W/L given in Equation (2), if the exponent e is in the neighborhood of 2.0.

Without specifying the value of *e*, substitution of Equation (2) in Equation (1) gives the following expression for the load, P_f, supported by the two flap panels:

$$P_f = c\, P_{mx}^{\ b} \left(\sqrt{D_x D_y}\right)^{1-b} (1 + W/L)^a\, W^{2b-1} \qquad (3)$$

where *a* may be regarded as an empirical constant [$a = e(1-b)$].

If the data of Table I, or Fig. 3, pertaining to the effect of cut-away inner flaps in cubical boxes are replotted in terms of the flap ratio of Equation (2), namely, $(1 + W/L)$, where W/L is in this case taken to be the ratio of double thickness board to flap-panel height, the curves of Fig. 4 are obtained. Except for the discontinuity in the curve near $(1 + W/L) = 2.0$ (that is, zero gap), the relationship between end-load box compression and $(1 + W/L)$ appears to be concave downward. This indicates that the exponent *a* in Equation (3) should be less than unity.

It should be mentioned that the justification for applying the ultimate plate strength equation to the outer flaps of a box in end-load is by no means certain. The method has been applied successfully to metallic plates with one lateral edge free and one lateral edge supported, and this resembles an outer flap of a corrugated box. However, with metallic plates, it has been observed that failure of the material initiates at the supported edge rather than at the free edge (5), whereas the situation is reversed in end-load compression. Also, the approximations which lead to the simplified expression (1) are based on experience with top-load compression and have not been examined critically for end-load. Moreover, no attempt has yet been made to establish the size limits on boxes for which the expression may apply, in contrast to the top-load analysis. Despite these uncertainties, the expression given above for the load, P_f, on the flap panel is believed to be a plausible first approach to the end-load analysis.

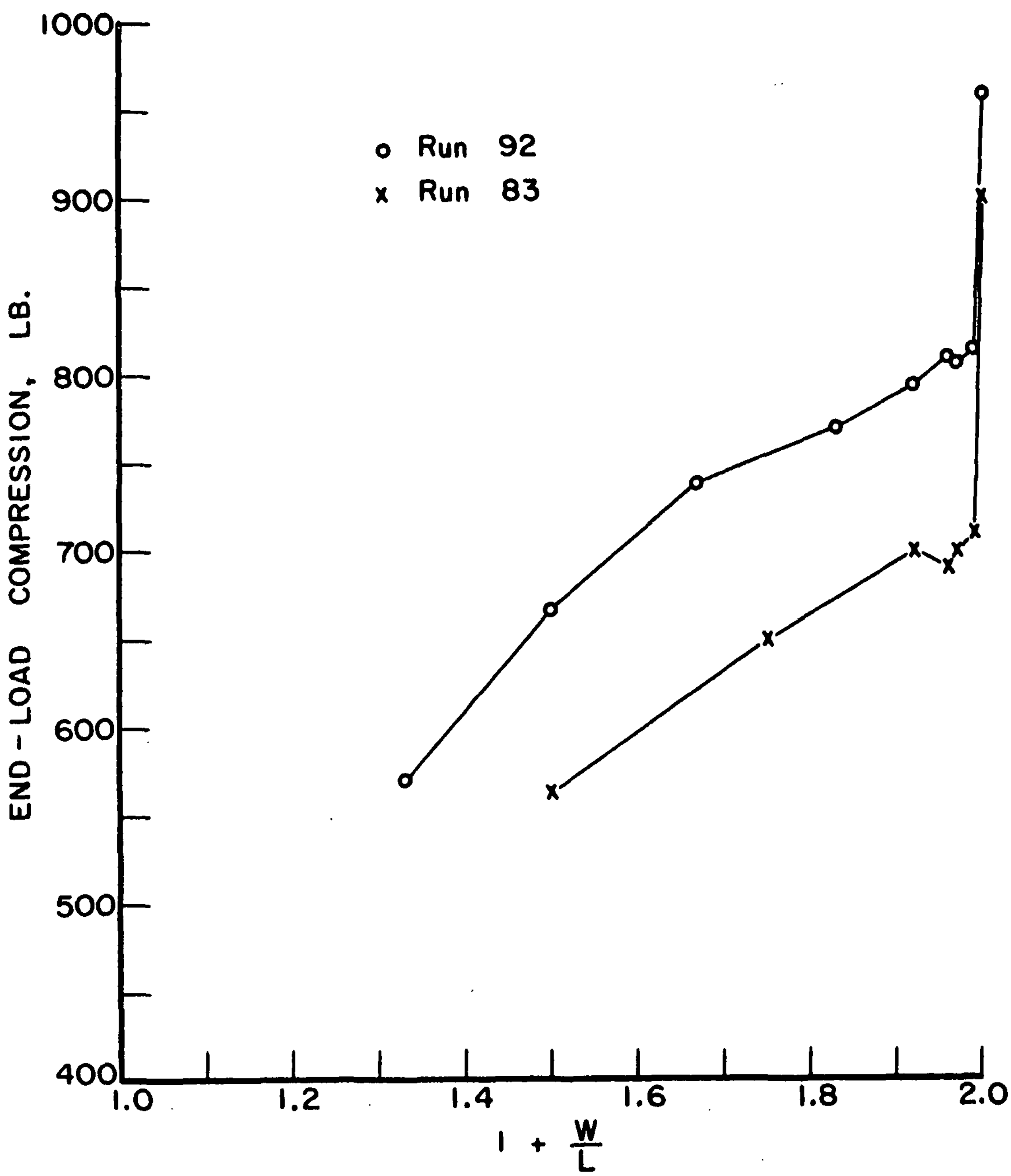

Figure 4. Relationship Between End-Load Compression and Panel Stiffness Factor (1 + W/L) for 12-Inch Cubical Boxes With Cut-Away Inner Flaps

Directing attention to the side panels, it may be appreciated that no simple property of the combined board or components is likely to adequately reflect the resistance of the scoreline to the crushing and rolling which occurs at the loading perimeter. While special tests on scored combined board probably could be devised, it is not anticipated that they would offer great utility in component manufacture.

In view of the aforementioned rather modest load contribution from the side panels, it appeared that even a crude approximation to the crushing and rolling resistance may not introduce prohibitive error in the estimate of total box load. It was assumed, therefore, that the rolling and crushing resistance of the combined board at the scoreline is proportional to its edgewise compression strength in the machine direction. It may be argued that somewhat similar mechanisms of liner buckling occur in both instances. With this assumption, the load, P_s, supported by the two side panels may be expressed as

$$P_s = 2\, f\, P_{mx}\, d \qquad (4)$$

where d = box depth, in.

f = an empirical constant (factor of proportionality between scoreline crushing and edgewise compression strength)

The total load supported by the box is $P = P_f + P_s$ and is given by the sum of Equations (3) and (4). In total, the end-load formula involves the three box dimensions (L, W, and d), the edgewise compression strength of the combined board in the machine direction (P_{mx}), the composite flexural stiffness of the combined board ($\sqrt{D_x D_y}$), and four empirical constants a, b, c, and f.

The empirical constants were determined so as to best fit the experimental data from 57 samples of boxes available from the commercial box study.

End-load compression tests were performed on five boxes from each sample. The composite flexural stiffness of the combined board was evaluated by the four-point beam method in connection with the top-load compression study. For evaluation of the M.D. edgewise compression strength of the combined board, short columns were tested, having dimensions 4-1/2 flutes high and two inches wide, and with the loading edges reinforced to a depth of one flute with Mobilwax D paraffin. Ten specimens were tested from each sample of board. The data are shown in Table II. It may be of interest to note that comparison of these short column loads with the sum of the M.D. modified ring strengths of the two liners indicated that the combined board does not exhibit the strength potential of the liners in true edgewise compression (3). The short column strength was only about 45%, on the average, of the sum of the ring strengths of the liner in the case of A-flute, 55% for C-flute, and 65% for B-flute. The magnitude and trend of these ratios is evidence of interflute buckling of the liners.

For the purposes of determining the empirical constants, the equation for total box load was written in the following form:

$$\log \left[\frac{P/W - 2f\, P_{mx}\, (d/W)}{\sqrt{D_x D_y}/W^2}\right] = \log c + b \log \left[\frac{P_m}{\sqrt{D_x D_y}/W^2}\right] + a \log\, [1 + W/L] \quad (5)$$

The computational procedure was as follows. A value of f was assumed, thereby permitting calculation of all the terms within brackets in Equation (5), whereupon Equation (5) has the following form:

$$\log Y = \log c + b \log X_1 + a \log X_2 \quad (6)$$

The values of log c, b, and a giving the best fit (for the assumed f) were determined by the method of least squares. With these values of a, b, c, and f, the end-load equation was employed to predict total load which was compared with the observed

TABLE II

PHYSICAL PROPERTIES OF COMBINED BOARD AND COMPARISON OF
ESTIMATED AND OBSERVED END-LOAD BOX COMPRESSION LOAD

Code	Series, lb.	Box Dimensions, in. Length, L	Width, W	Depth, d	Ratio W/L	Edgewise Compression Strength, P_{mx}, lb./in.	Composite Flexural Stiffness, $\sqrt{D_xD_y}$, lb.-in.	Box Load, lb. Observed	Estimated	Diff., %[a]
						A-Flute				
1192	175	9.75	7.50	7.75	0.769	13.0	121	342	306	-10.5
1188	175	12.50	10.75	15.75	.860	13.5	117	446	418	-5.9
2046	175	17.25	11.50	23.75	.667	19.2	134	617	612	-0.8
						Av. 15.2	124			
1176	200	14.50	7.75	14.75	.534	17.0	141	440	409	-7.2
1184	200	15.50	9.25	17.75	.600	16.4	129	472	447	-5.4
2324	200	13.50	13.50	17.25	1.000	23.3	131	691	783	+13.4
1146	200	14.00	14.00	14.00	1.000	17.8	128	631	620	-1.8
1167	200	16.50	15.25	6.75	.924	17.2	139	647	590	-8.7
1163	200	17.50	15.00	8.75	.857	20.0	130	699	653	-6.6
1172	200	19.50	13.50	11.50	.692	18.2	140	609	571	-6.2
2315	200	23.50	16.75	11.38	.713	21.9	119	724	721	-0.4
2303	200	29.00	20.50	10.25	.707	19.3	126	638	723	+13.3
						Av. 19.0	131			
1180[c]	275	9.75	6.50	14.75	.667	31.8	115	574	634	+10.4
1197[c]	275	9.50	9.50	12.50	1.000	31.0	125	756	787	+4.1
2041	275	11.50	8.75	16.25	.761	34.0	159	741	844	+14.0
2373	275	20.00	16.00	13.25	.800	34.5	190	1025	1157	+12.9
2095	275	24.38	19.00	15.12	.779	33.0	175	1258	1212	-3.7
2099	275	37.75	20.50	31.25	.543	43.1	171	1793	1671	-6.8
2054	275	36.00	24.00	31.00	.667	36.2	185	1563	1615	+3.3
2159	275	42.50	24.00	17.25	.565	41.7[b]	207	1633	1633	0.0
2155	275	36.75	30.25	28.25	.823	39.0	215	1955	2004	+2.5
						Av. 36.0	171		Av.	6.6[e]
						C-Flute				
2219	175	10.75	5.38	10.50	0.500	20.6	75.8	294	337	+14.7
2167	175	12.62	9.25	5.75	.733	16.9	70.7	365	367	+0.4
						Av. 18.8	73.2			
2211	200	10.25	10.00	12.25	.976	20.3	62.3	447	499	+11.7
2176	200	12.75	8.50	6.75	.667	23.0	81.5	358	461	+28.7
2107	200	12.50	9.25	10.25	.740	24.3	101	559	560	+0.2
2081	200	13.62	8.75	9.38	.642	23.5	96.0	466	506	+8.7
2228	200	15.50	8.00	10.50	.516	27.6	84.1	515	530	+2.8
2207	200	16.62	10.00	7.25	.602	20.4	78.3	522	446	-14.5
2076	200	16.62	10.12	7.75	.609	24.8	103	537	559	+4.0
2058	200	13.50	13.50	10.50	1.000	26.2	93.1	738	758	+2.7
2365	200	16.00	12.00	14.12	.750	22.9	73.8	551	602	+9.2
2215	200	17.00	11.75	11.25	.691	25.3	78.4	529	622	+17.6
2145	200	18.38	10.38	11.75	.565	23.1	84.3	513	536	+4.4
2171	200	17.50	11.62	7.25	.664	23.6	88.3	624	565	-9.4
2150	200	19.50	12.88	10.62	.660	23.6	78.2	667	606	-9.1
2090	200	18.25	18.12	16.25	.993	23.8	82.6	891	838	-6.0
2094	200	26.62	13.00	20.50	.488	21.8	81.0	711	611	-14.0
						Av. 23.6	84.4			
2111	275	12.50	12.38	12.25	.990	40.6	120	1004	1105	+10.1
2141	275	19.75	8.00	11.00	.405	47.0[b]	139	911	876	-3.9
2369	275	18.00	15.25	14.12	.847	41.8[b]	128	1403	1253	-10.7
2033	275	22.50	15.75	22.75	.700	38.9	116	1272	1237	-2.8
2050	275	21.50	18.25	16.00	.849	41.9	148	1489	1434	-3.7
2103	275	24.00	19.75	18.25	.823	40.6	108	1481	1391	-6.1
						Av. 41.8	126			
2137	350	11.00	11.00	17.00	1.000	51.0	126	1433[d]	1343	-6.3
									Av.	8.4[e]
						B-Flute				
2349	175	17.50	11.25	22.00	0.643	19.0	38.8	582	480	-17.5
2353	200	11.50	8.50	24.12	.739	26.3	48.4	687	611	-11.0
2311	200	13.50	11.25	13.12	.833	34.4[b]	48.0	818	762	-6.9
2009	200	15.62	10.44	14.62	.668	25.9	42.0	634	558	-12.0
2001	200	16.25	12.25	11.50	.754	25.4	41.3	533	578	+8.4
2029	200	17.50	11.25	21.75	.643	32.8	46.6	669	781	+16.7
2319	200	20.88	10.75	9.50	.515	32.2[b]	41.3	537	601	+11.8
2307	200	20.50	13.50	9.00	.658	34.1	52.7	759	761	+0.3
						Av. 30.2	45.8			
2062	275	12.00	12.00	12.00	1.000	50.9[b]	80.3	1468	1217	-17.1
2240	275	13.00	11.25	10.62	.865	52.7	82.3	1018	1159	+13.9
2182	275	17.25	10.00	22.50	.580	43.7[b]	79.5	1022	1024	+0.2
2248	275	21.12	15.50	18.62	.734	52.9	74.1	1242	1422	+14.5
						Av. 50.0	79.0		Av.	10.9[e]
								Composite Av.		8.25[e]

[a] Diff., % based on observed box load.
[b] Average of twenty specimens.
[c] Cylinder formed liners.
[d] Average of three specimens.

[e] Average absolute difference.

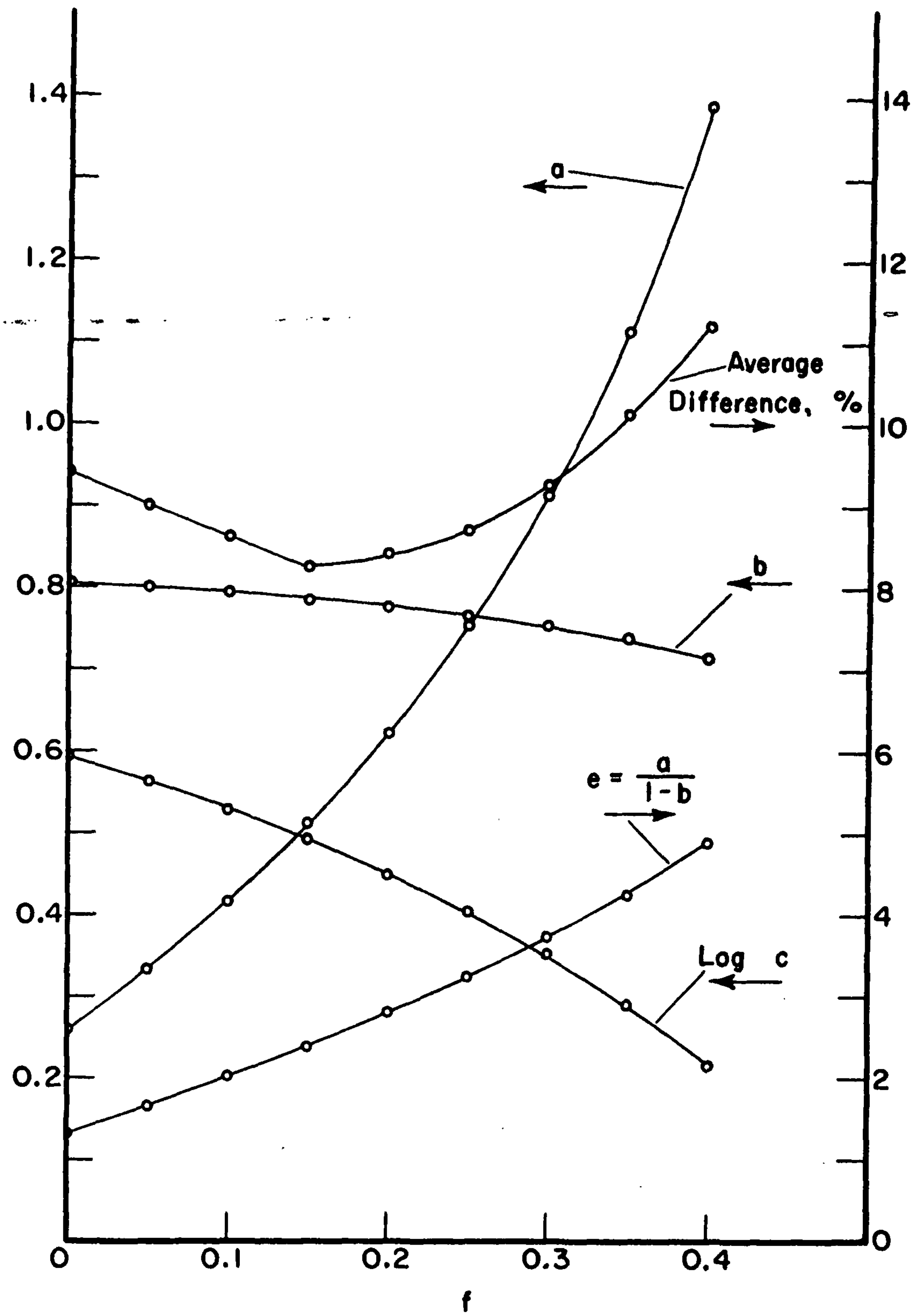

Figure 5. Variation of Average Difference and Empirical Constants as a Function of the Assigned Constant _f_

It may be seen in Table II that on the basis of averages the per cent difference was least for A-flute boxes and greatest for B-flute boxes, namely, 6.6% for A-flute, 8.4% for C-flute, and 10.9% for B-flute. There is no evidence, however, that the formula systematically over- or underestimates for a given flute size inasmuch as roughly half of the differences within a flute size are positive and the remaining half are negative.

While the accuracy of this end-load formula is not as favorable as with top-load [6.1%, on the average (1)], it is pertinent to note that the variability of end-load compression is substantially higher than that of top load; on the average, the standard deviations were 9.3 and 6.6% of the mean in the two types of tests, respectively. Because of this high variability, the end-load compression strength of these samples (five boxes per sample) is known to only about ± 11% (with 95% confidence), whereupon at least 60% of the differences between estimated and observed end load are not significant.

It should be mentioned that six of these 57 samples were "all-flaps-meet" boxes, that is, zero gap on the flap panels. Although the end-load strength of four of these six samples was predicted with good accuracy, consideration of Fig. 3 and 4 indicates that these zero gap boxes perhaps should be treated as a separate class because of the discontinuity in load at zero gap. That is, the empirical treatment of the effective stiffness of the flap panels $[(1 + W/L)^{a}]$ cannot be expected to apply at the point of discontinuity. Because the existence of the discontinuity was not known at the time that the numerical work leading to Equation (7) was performed, the six zero gap samples were included, and therefore they influence the empirical constants. While it would be desirable to repeat the computation (leading to the empirical constants) excluding the six zero gap box samples, this has been withheld pending the results of developmental studies on the methodology of the test for $\underline{P}_{mx}$; it is possible that improved evaluations of this property may be included in the recomputation which is quite lengthy.

It is of interest to examine the end-load box equation [Equation (7)] with respect to the salient characteristics of end-load compression behavior mentioned above. The relative performance of A- and B-flute boxes and the relative load-carrying ability of flap panels and side panels can be examined by means of a numerical example. Assume a box with dimensions $\underline{L} = 15$, $\underline{W} = 10$, and $\underline{d} = 10$ inches is fabricated in both A and B-flute combined board in the 200-lb. series and manufactured from the same components. The combined board properties will be assumed to be as follows, based on average values from Table II:

A-flute	B-flute
$\underline{P}_{mx} = 19.0$ lb./in.	$\underline{P}_{mx} = 30.2$ lb./in.
$\sqrt{\underline{D}_x\underline{D}_y} = 131$ lb.-in.	$\sqrt{\underline{D}_x\underline{D}_y} = 45.8$ lb.-in.

The computation of the estimated strength of the A-flute box is carried out in the following:

$$\begin{aligned}\underline{P}_A &= (0.3)(19.0)(10) + (3.10)(19.0)^{0.787}(131)^{0.213}(1.667)^{0.512}(10)^{0.574} \\ &= 57 + (3.10)(10.1)(2.82)(1.299)(3.75) \\ &= 57 + (3.10)(28.6)(1.299)(3.75) \\ &= 57 + 430 \\ &= 487 \text{ lb.}\end{aligned}$$

For B-flute:

$$\begin{aligned}\underline{P}_B &= (0.3)(30.2)(10) + (3.10)(30.2)^{0.787}(45.8)^{0.213}(1.667)^{0.512}(10)^{0.574} \\ &= 91 + (3.10)(14.6)(2.26)(1.299)(3.75)\end{aligned}$$

$= 91 + (3.10)(33.0)(1.299)(3.75)$

$= 91 + 497$

$= 588$ lb.

Thus, the B-flute box load is about 21% higher than the A-flute box load, which is in the direction and of typical magnitude of general experience. By following through the illustrative computation it may be seen that the higher flexural stiffness of the A-flute board is more than offset by the higher edgewise compression strength of B-flute board.

The first term in each computation is the contribution from the two side panels and the second term is from the flap panels. In the A-flute example, the side panel contribution is about 13% of the flap panel contribution; with B-flute the ratio is 18%. These contributions are in the expected direction but are lower than was experienced in load distribution measurements, namely, 25 to 33%, and lead one to question whether sufficient weight is given to the side panels by the empirically derived box equation.

With regard to the effect of box dimensions on end-load strength, Equation (7) indicates that load increases linearly with the box depth dimension, $\underline{d}$. This trend is as anticipated, of course, but data are not available to affirm the linear relationship—this being an assumption in formulating the box equation.

With regard to the box dimensions, $\underline{L}$ and $\underline{W}$, three distinct cases may be considered:

a. Suppose width, $\underline{W}$, and depth, $\underline{d}$, are held constant and length, $\underline{L}$, is increased. This causes an increase in the flap gap but no change in the loading perimeter, $2(\underline{W} + \underline{d})$, of the box. As regards the box equation, $\underline{W}/\underline{L}$ decreases, and therefore box load decreases, as anticipated.

b. Suppose length, L, and depth, d, are held constant, and width, W, is decreased. This variation of dimensions causes an increase in the gap and a decrease in the loading perimeter, both of which may be expected to lead to a decrease in box load. In terms of the equation, both W/L and W decrease, and there is clearly a decrease in the estimated box load.

c. Suppose both L and W are increased, but in such a manner that W/L (and depth, d) are held constant. This case corresponds to an increased gap (consider, for example, L = 15, W = 10, and L = 24, W=16, giving gaps of 5 and 8 inches, respectively) and an increase in loading perimeter, which have opposite effects on box load. The formula predicts an increase in box load, but less than proportional to the increase in W inasmuch as it enters to the 0.574 power. This would be an interesting case to verify experimentally. Unpublished data with end-load "tubes" followed the predicted trend.

In summary, the end-load box equation is in general agreement with the major characteristics of end-load box behavior as presently known.

FUTURE WORK

Although the accuracy capable of being obtained in estimating end-load box strength may be somewhat less than top load, it is believed that refinements in the theory of end-load strength should permit better accuracy than was achieved with Equation (7). It is recommended that further work should be directed to the following aspects of the problem:

1. Machine direction compression strength of combined board. It appears from the preliminary development of an end-load compression formula that the edgewise compression strength of the combined board is the dominant factor governing box strength (just as in the case of top-load compression). It is important, therefore, to have a confident means of evaluating this property. There is some indication that the short column test method employed in the preceding analysis could be improved. This belief stems from the observation that occasionally some of the specimens failed by rolling and crushing of a liner in the reinforcement zone at the loading edge, rather than by interflute buckling in the unreinforced zone as was desired.

Actually, little development work went into the current form of the machine-direction short column test, other than to ascertain that a reinforced-edge column offered a marked improvement over an unreinforced specimen in both load and specimen behavior. Work is now in progress to explore the effect of various means of reinforcing the edges. Four waxes with varying degrees of hardness, two thermoplastic resins and a household cement have been tried on a sample of A-flute, 200-lb. series board. Preliminary results indicate that one of the resins (MS-2, Howards and Sons, Cornwall, Ontario) offers a slight improvement over the earlier method in regard to load (5%) and gives consistent specimen behavior—that is, interflute buckling of the liners. Additional work is now underway to confirm these trends. A limited study is planned to explore the effect of specimen dimensions.

2. All-flaps-meet boxes. As mentioned earlier, physical considerations indicate that the analysis for an all-flaps-meet box may differ from those boxes having a gap between inner flaps. The load contribution from the inner flaps of an all-flaps-meet box is probably less than the cross-direction edgewise compression strength of the combined board because these flaps do not fail in the ordinary manner of edgewise compression and the butting edges of the inner flaps are usually damaged from the slitting operation. A special study may be required to determine the load contribution of the inner flaps.

3. Effective flexural stiffness of the flap panel. The effective flexural stiffness of the flap panels was treated in a highly empirical manner in the foregoing analysis. It is believed that a more rigorous analysis of this property as a function of the flap gap should be made. One approach to this objective may be bending tests of laminated boards with single-thickness "gaps" at the center of the span.

4. Buckling strength of single-face liner. As discussed above, there is question regarding the most effective method of evaluating the buckling strength of the single-face liner which triggers box failure. In addition to the studies of short column behavior mentioned above, an exploratory study with long columns is in progress. Possibly the behavior of this structural form can be related to the combined board in the flap gap and thereby provide an estimate of the strength of the combined board in end-load box compression. Another approach is to estimate the interflute buckling strength of the single-face liner from column theory (this would involve Taber stiffness of the liner). It is not entirely clear at this time as to the best method of experimentally verifying the buckling estimates

inasmuch as the load in a short column compression test on combined board apparently is the sum of the failure load of the single-face liner and a prefailure load from the double-face liner. An alternative may be to employ the liner buckling estimates in Equation (3) and determine whether an improvement is obtained in predicting end-load compression.

5. Study of the behavior of flap panels and side panels separately. It may be appreciated that one of the difficulties in analyzing end-load compression is that the total load on the box is the sum of the loads supported by two quite different types of panels, neither of which has been analyzed very extensively. It is believed that a technique which may be helpful for better understanding end-load compression is the cutting away of one or the other of the two types of panels in a given box, whereby the behavior of one type of panel may be studied more effectively. This type of study might also provide an opportunity to determine the effect of box dimensions on end-load, such as the effect of increasing L and W at constant W/L or the effect of variation in box depth, d.

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